

Package ‘ForestFit’

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Type Package

Title Statistical Modelling for Plant Size Distributions

Author Mahdi Teimouri

Maintainer Mahdi Teimouri <teimouri@aut.ac.ir>

Description Developed for the following tasks. 1) Computing the probability density function, cumulative distribution function, random generation, and estimating the parameters of the eleven mixture models. 2) Point estimation of the parameters of two - parameter Weibull distribution using twelve methods and three - parameter Weibull distribution using nine methods. 3) The Bayesian inference for the three - parameter Weibull distribution. 4) Estimating parameters of the three - parameter Birnbaum - Saunders, generalized exponential, and Weibull distributions fitted to grouped data using three methods including approximated maximum likelihood, expectation maximization, and maximum likelihood. 5) Estimating the parameters of the gamma, log-normal, and Weibull mixture models fitted to the grouped data through the EM algorithm, 6) Estimating parameters of the nonlinear height curve fitted to the height - diameter observation, 7) Estimating parameters, computing probability density function, cumulative distribution function, and generating realizations from gamma shape mixture model introduced by Venturini et al. (2008) <doi:10.1214/07-AOAS156> , 8) The Bayesian inference, computing probability density function, cumulative distribution function, and generating realizations from four - parameter Johnson SB distribution, and 9) Robust multiple linear regression analysis when error term follows skewed t distribution.

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DBH

*Trees height and diameter at breast height***Description**

The DBH data contains the diameter at breast height (dbh), height and condition data for all trees centered in 108 plots of size 0.2 hectare immediately following a single prescribed burn and also following three 5-yr (year) interval reburns (four burns total) and a single 15-yr interval reburn (two burns total) and associated treatment information. The trees information were established from mixed ponderosa pine (*Pinus ponderosa* Dougl. ex Laws.) that contained scattered western junipers (*Juniperus occidentalis* Hook.). The plots were located in the Malheur National Forest on the southern end of the Blue Mountains near Burns, Oregon, USA.

Usage

```
data(DBH)
```

Format

A text file with 5732 observations from 17 variables related of trees characteristics such as dbh and height.

References

B. K., Kerns, D. J., Westlind, and M. A. Day. 2017. Season and interval of burning and cattle exclusion in the southern blue mountains, oregon: Overstory tree height, diameter and growth. Forest Service Research Data Archive, <doi:10.2737/RDS-2017-0041> .

dgsm	<i>Computing probability density function of the gamma shape mixture model</i>
------	--

Description

Computes probability density function (pdf) of the gamma shape mixture (GSM) model. The general form for the pdf of the GSM model is given by

$$f(x, \Theta) = \sum_{j=1}^K \omega_j \frac{\beta^j}{\Gamma(j)} x^{j-1} \exp(-\beta x),$$

where $\Theta = (\omega_1, \dots, \omega_K, \beta)^T$ is the parameter vector and known constant K is the number of components. The vector of mixing parameters is given by $\omega = (\omega_1, \dots, \omega_K)^T$ where ω_j s sum to one, i.e., $\sum_{j=1}^K \omega_j = 1$. Here β is the rate parameter that is equal for all components.

Usage

```
dgsm(data, omega, beta, log = FALSE)
```

Arguments

data	Vector of observations.
omega	Vector of the mixing parameters.
beta	The rate parameter.
log	If TRUE, then log(pdf) is returned.

Value

A vector of the same length as data, giving the pdf of the GSM model.

Author(s)

Mahdi Teimouri

References

S. Venturini, F. Dominici, and G. Parmigiani, 2008. Gamma shape mixtures for heavy-tailed distributions, *The Annals of Applied Statistics*, 2(2), 756–776.

Examples

```
data<-seq(0,20,0.1)
omega<-c(0.05, 0.1, 0.15, 0.2, 0.25, 0.25)
beta<-2
dgsb(data, omega, beta)
```

djsb

Computing the probability density function of Johnson's SB (JSB) distribution

Description

Computes the probability density function of the four-parameter JSB distribution given by

$$f(x|\Theta) = \frac{\delta\lambda}{\sqrt{2\pi}(x-\xi)(\lambda+\xi-x)} \exp\left\{-\frac{1}{2}\left[\gamma + \delta \log\left(\frac{x-\xi}{\lambda+\xi-x}\right)\right]^2\right\},$$

where $\xi < x < \lambda + \xi$, $\Theta = (\delta, \gamma, \lambda, \xi)^T$ with $\delta, \lambda > 0$, $-\infty < \gamma < \infty$, and $-\infty < \xi < \infty$.

Usage

```
djsb(data, param, log = FALSE)
```

Arguments

data	Vector of observations.
param	Vector of the parameters δ , γ , λ , and ξ .
log	If TRUE, then log(pdf) is returned.

Value

A vector of length n, giving random generated values from JSB distribution.

Author(s)

Mahdi Teimouri

Examples

```
data<-rnorm(10)
param<-c(delta<-1, gamma<-3, lambda<-12, xi<-5)
djsb(data, param, log = FALSE)
```

dmixture

Computing probability density function of the well-known mixture models

Description

Computes probability density function (pdf) of the mixture model. The general form for the pdf of the mixture model is given by

$$f(x, \Theta) = \sum_{j=1}^K \omega_j f_j(x, \theta_j),$$

where $\Theta = (\theta_1, \dots, \theta_K)^T$, is the whole parameter vector, θ_j for $j = 1, \dots, K$ is the parameter space of the j -th component, i.e. $\theta_j = (\alpha_j, \beta_j)^T$, $f_j(\cdot, \theta_j)$ is the pdf of the j -th component, and known constant K is the number of components. The vector of mixing parameters is given by $\omega = (\omega_1, \dots, \omega_K)^T$ where ω_j s sum to one, i.e., $\sum_{j=1}^K \omega_j = 1$. Parameters α_j and β_j are the shape and scale parameters of the j -th component or both are the shape parameters. In the latter case, the parameters α and β are called the first and second shape parameters, respectively. We note that the constants ω_j s sum to one, i.e. $\sum_{j=1}^K \omega_j = 1$. The families considered for each component include Birnbaum-Saunders, Burr type XII, Chen, F, Frechet, Gamma, Gompertz, Log-normal, Log-logistic, Lomax, skew-normal, and Weibull with pdf given by the following.

- Birnbaum-Saunders

$$f(x, \theta) = \frac{\sqrt{\frac{x}{\beta}} + \sqrt{\frac{\beta}{x}}}{2\alpha x} \phi\left(\frac{\sqrt{\frac{x}{\beta}} - \sqrt{\frac{\beta}{x}}}{\alpha}\right),$$

- Burr XII

$$f(x, \theta) = \alpha \beta x^{\alpha-1} (1 + x^\alpha)^{-\beta-1},$$

- Chen

$$f(x, \theta) = \alpha \beta x^\alpha \exp(x^\alpha) \exp\{-\beta \exp(x^\alpha) + \beta\},$$

- F

$$f(x, \theta) = \frac{\Gamma\left(\frac{\alpha+\beta}{2}\right)}{\Gamma\left(\frac{\alpha}{2}\right)\Gamma\left(\frac{\beta}{2}\right)} \left(\frac{\alpha}{\beta}\right)^{\frac{\alpha}{2}} x^{\frac{\alpha}{2}-1} \left(1 + \frac{\alpha}{\beta}x\right)^{-\frac{\alpha+\beta}{2}},$$

- Frechet

$$f(x, \theta) = \frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{-\alpha-1} \exp\left\{-\left(\frac{x}{\beta}\right)^{-\alpha}\right\},$$

- gamma

$$f(x, \theta) = [\beta^\alpha \Gamma(\alpha)]^{-1} x^{\alpha-1} \exp\left(-\frac{x}{\beta}\right),$$

- Gompertz

$$f(x, \theta) = \beta \exp(\alpha x) \exp\left\{\frac{\beta \exp(\alpha x) - 1}{\alpha}\right\},$$

- log-logistic

$$f(x, \theta) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} \left[\left(\frac{x}{\beta} \right)^\alpha + 1 \right]^{-2},$$

- log-normal

$$f(x, \theta) = (\sqrt{2\pi}\beta x)^{-1} \exp\left\{-\frac{1}{2} \left(\frac{\log x - \alpha}{\beta} \right)^2\right\},$$

- Lomax

$$f(x, \theta) = \frac{\alpha\beta}{(1 + \alpha x)^{\beta+1}},$$

- skew-normal

$$f(x, \theta) = 2\phi\left(\frac{x - \alpha}{\beta}\right)\Phi\left(\lambda\frac{x - \alpha}{\beta}\right),$$

- Weibull

$$f(x, \theta) = \frac{\alpha}{\beta} \left(\frac{x}{\beta} \right)^{\alpha-1} \exp\left\{-\left(\frac{x}{\beta}\right)^\alpha\right\},$$

where $\theta = (\alpha, \beta)$. In the skew-normal case, $\phi(\cdot)$ and $\Phi(\cdot)$ are the density and distribution functions of the standard normal distribution, respectively.

Usage

```
dmixture(data, g, K, param)
```

Arguments

data	Vector of observations.
g	Name of the family including "birnbaum-saunders", "burrxii", "chen", "f", "Frechet", "gamma", "gompertz", "log-normal", "log-logistic", "lomax", "skew-normal", and "weibull".
K	Number of components.
param	Vector of the ω , α , β , and λ .

Details

For the skew-normal case, α , β , and λ are the location, scale, and skewness parameters, respectively.

Value

A vector of the same length as data, giving the pdf of the mixture model of families computed at data.

Author(s)

Mahdi Teimouri

Examples

```

data<-seq(0,20,0.1)
K<-2
weight<-c(0.6,0.4)
alpha<-c(1,2)
beta<-c(2,1)
param<-c(weight,alpha,beta)
dmixture(data, "weibull", K, param)

```

fitbayesJSB

Estimating parameters of the Johnson's SB (JSB) distribution using the Bayesian approach

Description

Suppose $x = (x_1, \dots, x_n)^T$ denotes a vector of n independent observations coming from a four-parameter JSB distribution with probability density function given by

$$f(x|\Theta) = \frac{\delta\lambda}{\sqrt{2\pi}(x-\xi)(\lambda+\xi-x)} \exp\left\{-\frac{1}{2}\left[\gamma + \delta \log\left(\frac{x-\xi}{\lambda+\xi-x}\right)\right]^2\right\},$$

where $\xi < x < \lambda + \xi$, $\Theta = (\delta, \gamma, \lambda, \xi)^T$ with $\delta, \lambda > 0$, $-\infty < \gamma < \infty$, and $-\infty < \xi < \infty$. Using the Bayesian approach, we compute the Bayes' estimators of the JSB distribution parameters.

Usage

```
fitbayesJSB(data, n.burn=8000, n.simul=10000)
```

Arguments

data	Vector of observations.
n.burn	Length of the burn-in period, i.e., the point after which Gibbs sampler is supposed to attain convergence. By default n.burn is 8000.
n.simul	Total numbers of Gibbs sampler iterations. By default n.simul is 10,000.

Details

The Bayes' estimators are obtained by averaging on the all iterations between n.burn and n.simul.

Value

A list of objects in two parts as

1. Bayes' estimators of the parameters.
2. A sequence of four goodness-of-fit measures consist of Anderson-Darling (AD), Cram\`eer-von Misses (CVM), Kolmogorov-Smirnov (KS), and log-likelihood (log-likelihood) statistics.

Author(s)

Mahdi Teimouri

References

N. L. Johnson, 1949. Systems of frequency curves generated by methods of translation, *Biometrika*, 36, 149–176.

L. J. Norman, S. Kotz, and N. Balakrishnan, 1994. *Continuous Univariate Distributions*, volume I, John Wiley & Sons.

Examples

```
# Here we use the SW dataset provided by FIA that represents a typical loblolly pine plantation.
# As the variable of interest, we fit the JSB ditribution to the diameter at breast height (SW$DIA)
# in inches.
data(SW)
data<-SW$DIA
fitbayesJSB(data, n.burn=20, n.simul=50)
```

fitbayesWeibull	<i>Estimating parameters of the Weibull distribution using the Bayesian approach</i>
-----------------	--

Description

Suppose $x = (x_1, \dots, x_n)^T$ denotes a vector of n independent observations coming from a three-parameter Weibull distribution. Using the methodology given in Green et al. (1994), we compute the Bayes' estimators of the shape, scale, and location parameters.

Usage

```
fitbayesWeibull(data, n.burn=8000, n.simul=10000)
```

Arguments

data	Vector of observations.
n.burn	Length of the burn-in period, i.e., the point after which Gibbs sampler is supposed to attain convergence. By default n.burn is 8000.
n.simul	Total numbers of Gibbas sampler iterations. By default n.simul is 10,000.

Details

The Bayes' estimators are obtained by averaging on the all iterations between n.burn and n.simul.

Value

A list of objects in two parts as

1. Bayes' estimators of the parameters.
2. A sequence of four goodness-of-fit measures consist of Anderson-Darling (AD), Cram\`eer-von Misses (CVM), Kolmogorov-Smirnov (KS), and log-likelihood (log-likelihood) statistics.

Note

The methodology used here for computing the Bayes' estimator of the location parameter is different from that used by Green et al. (1994). This means that the location parameter is allowed to be any real value.

Author(s)

Mahdi Teimouri

References

E. J. Green, F. A. R. Jr, A. F. M. Smith, and W. E. Strawderman, 1994. Bayesian estimation for the three-parameter Weibull distribution with tree diameter data, *Biometrics*, 50(1), 254-269.

Examples

```
n<-20
alpha<-2
beta<-2
theta<-3
data<-rweibull(n,shape=alpha,scale=beta)+theta
fitbayesWeibull(data, n.burn=20, n.simul=50)
```

fitcurve

Estimatinng the parameters of the nonlinear curve fitted to the height-diameter(H-D) observations

Description

Estimates the parameters of the nine well-known nine three-parameter nonlinear curves fitted to the height-diameter observations. These nine models are given by the following.

- Richards (Richards(1959))

$$H = 1.3 + \beta_1 + \frac{\beta_2}{D + \beta_3},$$

- Gompertz (Winsor(1992))

$$H = 1.3 + \beta_1 e^{-\beta_2 e^{-\beta_3 D}},$$

- Hossfeld IV (Zeide(1993))

$$H = 1.3 + \frac{\beta_1}{1 + \frac{1}{\beta_2 D^{\beta_3}}},$$

- Korf (Flewelling and De Jong(1994))

$$H = 1.3 + \beta_1 e^{-\beta_2 D^{-\beta_3}},$$

- logistic (Pearl and Reed (1920))

$$H = 1.3 + \frac{\beta_1}{1 + \beta_2 e^{-\beta_3 D}},$$

- Prodan (Prodan(1968))

$$H = 1.3 + \frac{D^2}{\beta_1 D^2 + \beta_2 D + \beta_3},$$

- Ratkowsky (Ratkowsky(1990))

$$H = 1.3 + \beta_1 e^{-\frac{\beta_2}{D + \beta_3}},$$

- Sibbesen (Huang et al. (1992))

$$H = 1.3 + \beta_1 D^{\beta_2 D^{-\beta_3}},$$

- Weibull (Yang et al. (1978))

$$H = 1.3 + \beta_1 \left(1 - e^{-\beta_2 D^{\beta_3}}\right),$$

Usage

fitcurve(h,d,model,starts)

Arguments

h	Vector of height observations.
d	Vector of diameter observations.
model	The name of the fitted model including "chapman-richards", "gompertz", "hossfeldiv", "korf", "logistic", "prodan", "ratkowsky", "Sibbesen", and "weibull".
starts	A list of starting values for the parameters β_1 , β_2 , and β_3 .

Value

A list of objects in four parts as

1. Estimated parameters and corresponding summaries including standard errors, computed t -statistics, and p -values.
2. Residuals.
3. Covariance matrix of the estimated model parameters (coefficients) $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$.
4. Residual standard error, i.e., $\hat{\sigma}$.
5. The hieght-diameter scatterplot superimposed by the fitted model.

Author(s)

Mahdi Teimouri

References

- J. W. Flewelling and R. De Jong. (1994). Considerations in simultaneous curve fitting for repeated height-diameter measurements, *Canadian Journal of Forest Research*, 24(7), 1408-1414.
- S. Huang, S. J. Titus, and D. P. Wiens. 1992. Comparison of nonlinear height±diameter functions for major Alberta tree species. *Canadian Journal of Forest Research*, 22, 1297-1304.
- R. Pearl and L. J. Reed. (1920). On the rate of growth of the population of the United States since 1790 and its mathematical representation, *Proceedings of the National Academy of Sciences of the United States of America*, 6(6), 275.
- M. Prodan. 1968. The spatial distribution of trees in an area. *Allg. Forst Jagdztg*, 139, 214-217.
- D. A. Ratkowsky. 1990. *Handbook of nonlinear regression*, New York, Marcel Dekker, Inc.
- F. J. Richards. 1959. A flexible growth function for empirical use. *Journal of Experimental Botany*, 10, 290-300.
- S. B. Winsor. 1992. The Gompertz curve as a growth curve. *Proceedings of National Academic Science*, USA, 18, 1-8.
- R. C. Yang, A. Kozak, J. H. G. Smith. 1978. The potential of Weibull-type functions as a flexible growth curves. *Canadian Journal of Forest Research*, 8, 424-431.
- B. Zeide. 1993. Analysis of growth equation. *Forest Science*, 39, 594-616.

Examples

```
# use the heigth and diameter at breast height (dbh) of the plot 55 in DBH data set.
# The first column of DBH dataset contains the plot number. Also, H and D denote the
# height and dbh variables that located at columns 10 and 11 of data set DBH, respectively.
data(DBH)
D<-DBH[DBH[,1]==55,10]
H<-DBH[DBH[,1]==55,11]
starts<-c(9,0.0005,2)
fitcurve(H,D,"weibull",starts=starts)
```

fitgrouped

Estimating parameters of the three-parameter Birnbaum-saunders (BS), generalized exponential (GE), and Weibull distributions fitted to grouped data

Description

Suppose a sample of n independent observations each follows a three-parameter BS, GE, or Weibull distributions have been divided into m separate groups of the form $(r_{i-1}, r_i]$, for $i = 1, \dots, m$. So, the likelihood function is given by

$$L(\Theta) = \frac{n!}{f_1! f_2! \dots f_m!} \prod_{i=1}^m [F(r_i|\Theta) - F(r_{i-1}|\Theta)]^{f_i},$$

where the r_0 is the lower bound of the first group, r_m is the upper bound of the last group, and f_i is the frequency of observations within i -th group provided that $n = \sum_{i=1}^m f_i$. The cdf of a three-parameter BS, GE, and Weibull distributions are given by

$$F(x; \Theta) = \left(1 - \exp\{-\beta(x - \mu)\}\right)^\alpha,$$

$$F(x; \Theta) = \Phi\left(\frac{\sqrt{\frac{x}{\beta}} - \sqrt{\frac{\beta}{x}}}{\alpha}\right),$$

and

$$F(x; \Theta) = 1 - \exp\left\{-\left(\frac{x - \mu}{\beta}\right)^\alpha\right\},$$

where $\Theta = (\alpha, \beta, \mu)^T$.

Usage

```
fitgrouped(r, f, family, method1, starts, method2)
```

Arguments

<code>r</code>	A numeric vector of length $m + 1$. The first element of r is lower bound of the first group and other m elements are upper bound of the m groups. We note that upper bound of the $(i - 1)$ -th group is the lower bound of the i -th group, for $i = 2, \dots, m$. The lower bound of the first group and upper bound of the m -th group are chosen arbitrarily.
<code>f</code>	A numeric vector of length m containing the group's frequency.
<code>family</code>	Can be either "birnbaum-saunders", "ge", or "weibull".
<code>method1</code>	A character string determining the method of estimation. It can be one of "aml", "em" and "ml". The short forms "aml", "em", and "ml" are described as follows. "aml" (for method of approximated maximum likelihood (aml)), "em" (for method of expectation maximization (em)), and "ml" (for method of maximum likelihood (ml)).
<code>starts</code>	A numeric vector of the initial values for the shape, scale, and location parameters, respectively.
<code>method2</code>	The method for optimizing the log-likelihood function. It involves one of "BFGS", "Nelder-Mead", "CG", "L-BFGS-B" or "SANN".

Details

If the method is "em", then the initial values ("starts") and the log-likelihood optimizing method ("method2") are ignored.

Value

A two-part list of objects given by the following:

1. Estimated parameters of the three-parameter GE, Birnbaum-Saunders, or Weibull distribution fitted to the grouped data.
2. A sequence of goodness-of-fit measures consist of Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Anderson-Darling (AD), Chi-square (Chi-square), Cram\`eer-von Misses (CVM), Kolmogorov-Smirnov (KS), and log-likelihood (log-likelihood) statistics.

Author(s)

Mahdi Teimouri

References

- G. J. McLachlan and T. Krishnan, 2007. *The EM Algorithm and Extensions*, John Wiley & Sons.
- A. P. Dempster, N. M. Laird, and D. B. Rubin, 1977. Maximum likelihood from incomplete data via the EM algorithm, *Journal of the Royal Statistical Society, Series B (methodological)*, 1-38.
- M. Teimouri and A. K. Gupta, 2012. Estimation Methods for the Gompertz–Makeham Distribution Under Progressively Type-I Interval Censoring Scheme, *National Academy Science Letters*, 35(3).

Examples

```
r<-c(0,1,2,3,4,10)
f<-c(2,8,12,15,4)
starts<-c(2,2,0)
fitgrouped(r,f,"birnbaum-saunders","em")
fitgrouped(r,f,"weibull","ml",starts,"CG")
fitgrouped(r,f,"ge","em")
```

Description

Estimates parameters of the gamma shape mixture (GSM) model whose probability density function gets the form as follows.

$$f(x, \Theta) = \sum_{j=1}^K \omega_j \frac{\beta^j}{\Gamma(j)} x^{j-1} \exp(-\beta x),$$

where $\Theta = (\omega_1, \dots, \omega_K, \beta)^T$ is the parameter vector and known constant K is the number of components. The vector of mixing parameters is given by $\omega = (\omega_1, \dots, \omega_K)^T$ where ω_j s sum to one, i.e., $\sum_{j=1}^K \omega_j = 1$. Here β is the rate parameter that is equal for all components.

Usage

```
fitgsm(data,K)
```

Arguments

data	Vector of observations.
K	Number of components.

Details

Supposing that the number of components, i.e., K is known, the parameters are estimated through the EM algorithm developed by the maintainer.

Value

A list of objects in three parts as

1. The EM estimator of the rate parameter.
2. The EM estimator of the mixing parameters.
3. A sequence of goodness-of-fit measures consist of Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Anderson-Darling (AD), Cram\`eer-von Misses (CVM), Kolmogorov-Smirnov (KS), and log-likelihood (log-likelihood) statistics.

Author(s)

Mahdi Teimouri

References

- A. P. Dempster, N. M. Laird, and D. B. Rubin, 1977. Maximum likelihood from incomplete data via the EM algorithm, *Journal of the Royal Statistical Society Series B*, 39, 1-38.
- S. Venturini, F. Dominici, and G. Parmigiani, 2008. Gamma shape mixtures for heavy-tailed distributions, *The Annals of Applied Statistics*, 2(2), 756-776.

Examples

```
n<-100
omega<-c(0.05, 0.1, 0.15, 0.2, 0.25, 0.25)
beta<-2
data<-rgsm(n,omega,beta)
K<-length(omega)
fitgsm(data,K)
```

fitmixture

*Estimating parameters of the well-known mixture models***Description**

Estimates parameters of the mixture model using the expectation maximization (EM) algorithm. General form for the cdf of a statistical mixture model is given by

$$F(x, \Theta) = \sum_{j=1}^K \omega_j F_j(x, \theta_j),$$

where $\Theta = (\theta_1, \dots, \theta_K)^T$, is the whole parameter vector, θ_j for $j = 1, \dots, K$ is the parameter space of the j -th component, i.e. $\theta_j = (\alpha_j, \beta_j)^T$, $F_j(\cdot, \theta_j)$ is the cdf of the j -th component, and known constant K is the number of components. Parameters α and β are the shape and scale parameters or both are the shape parameters. In the latter case, the parameters α and β are called the first and second shape parameters, respectively. We note that the constants ω_j s sum to one, i.e. $\sum_{j=1}^K \omega_j = 1$. The families considered for the cdf F include Birnbaum-Saunders, Burr type XII, Chen, F, Frechet, Gamma, Gompertz, Log-normal, Log-logistic, Lomax, skew-normal, and Weibull.

Usage

```
fitmixture(data, family, K, initial=FALSE, starts)
```

Arguments

data	Vector of observations.
family	Name of the family including: "birnbaum-saunders", "burrxii", "chen", "f", "Frechet", "gamma", "gompertz", "log-normal", "log-logistic", "lomax", "skew-normal", and "weibull".
K	Number of components.
initial	The sequence of initial values including $\omega_1, \dots, \omega_K, \alpha_1, \dots, \alpha_K, \beta_1, \dots, \beta_K$. For skew normal case the vector of initial values of skewness parameters will be added. By default the initial values automatically is determined by k-means method of clustering.
starts	If initial=TRUE, then sequence of the initial values must be given.

Details

It is worth noting that identifiability of the mixture models supposed to be held. For skew-normal case we have $\theta_j = (\alpha_j, \beta_j, \lambda_j)^T$ in which $-\infty < \alpha_j < \infty$, $\beta_j > 0$, and $-\infty < \lambda_j < \infty$, respectively, are the location, scale, and skewness parameters of the j -th component, see Azzalini (1985).

Value

1. The output has three parts, The first part includes vector of estimated weight, shape, and scale parameters.
2. The second part involves a sequence of goodness-of-fit measures consist of Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Anderson-Darling (AD), Cram'eer-von Misses (CVM), Kolmogorov-Smirnov (KS), and log-likelihood (log-likelihood) statistics.
3. The last part of the output contains clustering vector.

Author(s)

Mahdi Teimouri

References

- A. Azzalini, 1985. A class of distributions which includes the normal ones, *Scandinavian Journal of Statistics*, 12, 171-178.
- A. P. Dempster, N. M. Laird, and D. B. Rubin, 1977. Maximum likelihood from incomplete data via the EM algorithm, *Journal of the Royal Statistical Society Series B*, 39, 1-38.
- M. Teimouri, S. Rezakhah, and A. Mohammadpour, 2018. EM algorithm for symmetric stable mixture model, *Communications in Statistics-Simulation and Computation*, 47(2), 582-604.

Examples

```
# Here we model the northern hardwood uneven-age forest data (HW$DIA) in inches using a
# 3-component Weibull mixture distribution.
data(HW)
data<-HW$DIA
K<-3
fitmixture(data,"weibull", K, initial=FALSE)
```


Description

Estimates parameters of the gamma, log-normal, and Weibull mixture models fitted to the grouped data using the expectation maximization (EM) algorithm. General form for the cdf of a statistical mixture model is given by

$$F(x, \Theta) = \sum_{k=1}^K \omega_k F_k(x, \theta_k),$$

where $\Theta = (\theta_1, \dots, \theta_K)^T$, is the whole parameter vector, θ_k for $k = 1, \dots, K$ is the parameter space of the j -th component, i.e. $\theta_k = (\alpha_k, \beta_k)^T$, $F_j(\cdot, \theta_j)$ is the cdf of the k -th component, and known constant K is the number of components. Parameters α and β are the shape and scale parameters. The constants ω_k s sum to one, i.e. $\sum_{k=1}^K \omega_k = 1$. The families considered for the cdf F include Gamma, Log-normal, and Weibull. If a sample of n independent observations each follows a distribution with cdf F have been divided into m separate groups of the form $(r_{i-1}, r_i]$, for $i = 1, \dots, m$. So, the likelihood function of the observed data is given by

$$L(\Theta | f_1, \dots, f_m) = \frac{n!}{f_1! f_2! \dots f_m!} \prod_{i=1}^m \left[\frac{F_i(\Theta)}{F(\Theta)} \right]^{f_i},$$

where

$$F_i(\Theta) = \sum_{k=1}^K \omega_k \int_{r_{i-1}}^{r_i} f(x|\theta_k) dx,$$

$$F(\Theta) = \sum_{k=1}^K \omega_k \int f(x|\theta_k) dx,$$

in which $f(x|\theta_k)$ denotes the pdf of the j -th component. Using the the EM algorithm proposed by Dempster et al. (1977), we can solve $\partial L(\Theta | f_1, \dots, f_m) / \partial \Theta = 0$ by introducing two new missing variables.

Usage

```
fitmixturegrouped(family, r, f, K, initial=FALSE, starts)
```

Arguments

family	Name of the family including: "gamma", "log-normal", "skew-normal", and "weibull".
r	A numeric vector of length $m + 1$. The first element of r is lower bound of the first group and other m elements are upper bound of the m groups. We note that upper bound of the $(i - 1)$ -th group is the lower bound of the i -th group, for $i = 2, \dots, m$. The lower bound of the first group and upper bound of the m -th group are chosen arbitrarily. If raw data are available, the smallest and largest observations are chosen for lower bound of the first group and upper bound of the m -th group, respectively.
f	A numeric vector of length m containing the group's frequency.
K	Number of components.

initial	The sequence of initial values including $\omega_1, \dots, \omega_K, \alpha_1, \dots, \alpha_K, \beta_1, \dots, \beta_K$. For skew normal case the vector of initial values of skewness parameters will be added. By default the initial values automatically is determined by k-means method of clustering.
starts	If initial=TRUE, then sequence of the initial values must be given.

Details

Identifiability of the mixture models supposed to be held. For skew-normal mixture model the parameter vector of k -th component gets the form $\theta_k = (\alpha_k, \beta_k, \lambda_k)^T$ where α_k, β_k , and λ_k denote the location, scale, and skewness parameters, respectively.

Value

1. The output has two parts, The first part includes vector of estimated weight, shape, and scale parameters.
2. A sequence of goodness-of-fit measures consist of Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Anderson-Darling (AD), Cramér-von Mises (CVM), Kolmogorov-Smirnov (KS), and log-likelihood (log-likelihood) statistics.

Author(s)

Mahdi Teimouri

References

G. J. McLachlan and P. N. Jones, 1988. Fitting mixture models to grouped and truncated data via the EM algorithm, *Biometrics*, 44, 571-578

Examples

```
n<-50
K<-2
m<-10
weight<-c(0.3,0.7)
alpha<-c(1,2)
beta<-c(2,1)
param<-c(weight,alpha,beta)
data<-rmixture(n, "weibull", K, param)
r<-seq(min(data),max(data),length=m+1)
D<-data.frame(table(cut(data,r,labels=NULL,include.lowest=TRUE,right=FALSE,dig.lab=4)))
f<-D$Freq
fitmixturegrouped("weibull",r,f,K,initial=FALSE)
```

fitWeibull	<i>Estimating parameters of the Weibull distribution through classical methods</i>
------------	--

Description

Estimates the parameters of the two- and three-parameter Weibull model with pdf and cdf given by

$$f(x; \alpha, \beta, \theta) = \frac{\alpha}{\beta} \left(\frac{x - \theta}{\beta} \right)^{\alpha-1} \exp \left\{ - \left(\frac{x - \theta}{\beta} \right)^\alpha \right\},$$

and

$$F(x; \alpha, \beta, \theta) = 1 - \exp \left\{ - \left(\frac{x - \theta}{\beta} \right)^\alpha \right\},$$

where $x > \theta$, $\alpha > 0$, $\beta > 0$ and $-\infty < \theta < \infty$. Here, the parameters α , β , and θ are known in the literature as the shape, scale, and location, respectively. If $\theta = 0$, then $f(x; \alpha, \beta)$ and $F(x; \alpha, \beta)$ in above are the pdf and cdf of a two-parameter Weibull distribution, respectively.

Usage

```
fitWeibull(data, location, method, starts)
```

Arguments

data	Vector of observations
starts	Initial values for starting the iterative procedures such as Newton-Raphson.
location	Either TRUE or FALSE. If location=TRUE, then shift parameter will be considered; otherwise the shift parameter omitted.
method	Used method for estimating the parameters. In the two-parameter case, methods are "greg1" (for the method of generalized regression type 1), "greg2" (for the method of generalized regression type 2), "lm" (for the method of L-moment), "ml" (for the method of maximum likelihood (ML)), "mlm" (for the method of logarithmic moment), "moment" (for the method of moment), "pm" (for the method of percentile), "rank" (for the method of rank correlation), "reg" (for the method of least square), "ustat" (for the method of U-statistic), "wml" (for the method of weighted ML), and "wreg" (for the method of weighted least square). In three-parameter case the methods are "mle" (for the method of ML), "mm1" (for the method of modified moment (MM) type 1), "mm2" (for the method of MM type 2), "mm3" (for the method of MM type 3), "mm11" (for the method of modified ML type 1), "mm12" (for the method of modified ML type 2), "mm13" (for the method of modified ML type 3), "mm14" (for the method of modified ML type 4), "moment" (for the method of moment), "mps" (for the method of maximum product spacing), "t1m" (for the method of T-L moment), and "wml" (for the method of weighted ML).

Details

For the method `wml`, all weights have been provided for sample size less than or equal to 100. This means that both methods `m1` and `wml` give the same estimates for samples of size larger than 100.

Value

A list of objects in two parts given by the following:

1. Estimated parameters for two- or three-parameter Weibull distribution.
2. A sequence of goodness-of-fit measures consist of Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Anderson-Darling (AD), Cram'eer-von Misses (CVM), Kolmogorov-Smirnov (KS), and log-likelihood (log-likelihood) statistics.

Author(s)

Mahdi Teimouri

References

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- F. Wang and J. B. Keats, 1995. Improved percentile estimation for the two-parameter Weibull distribution, *Microelectronics Reliability*, 35(6), 883-892.
- L. Zhang, M. Xie, and L. Tang, 2008. On Weighted Least Squares Estimation for the Parameters of Weibull Distribution. In: *Pham H. (eds) Recent Advances in Reliability and Quality in Design*. Springer Series in Reliability Engineering. Springer, London.

Examples

```

n<-100
alpha<-2
beta<-2
theta<-3
data<-rweibull(n,shape=alpha,scale=beta)+theta
starts<-c(2,2,3)
fitWeibull(data, TRUE, "mps", starts)
fitWeibull(data, TRUE, "wml", starts)
fitWeibull(data, FALSE, "mlm", starts)
fitWeibull(data, FALSE, "ustat", starts)

```

HW

*Mixed norther hardwood***Description**

Tree list from a U.S. Forest Service Forest Inventory and Analysis (FIA) plot PLT_CN 247006253010661 measured in 2012 and represents a typical northern hardwood uneven-age forest.

Usage

```
data(HW)
```

Format

A data frame containing 25 trees (rows) and two columns. Columns are the trees' scientific name and diameter at breast height in inches.

pgsm

*Computing cumulative distribution function of the gamma shape mixture model***Description**

Computes cumulative distribution function (cdf) of the gamma shape mixture (GSM) model. The general form for the cdf of the GSM model is given by

$$F(x, \Theta) = \sum_{j=1}^K \omega_j F(x, j, \beta),$$

where

$$F(x, j, \beta) = \int_0^x \frac{\beta^j}{\Gamma(j)} y^{j-1} \exp(-\beta y) dy,$$

in which $\Theta = (\omega_1, \dots, \omega_K, \beta)^T$ is the parameter vector and known constant K is the number of components. The vector of mixing parameters is given by $\omega = (\omega_1, \dots, \omega_K)^T$ where ω_j s sum to one, i.e., $\sum_{j=1}^K \omega_j = 1$. Here β is the rate parameter that is equal for all components.

Usage

```
pgsm(data, omega, beta, log.p = FALSE, lower.tail = TRUE)
```

Arguments

data	Vector of observations.
omega	Vector of the mixing parameters.
beta	The rate parameter.
log.p	If TRUE, then log(cdf) is returned.
lower.tail	If FALSE, then 1-cdf is returned.

Value

A vector of the same length as data, giving the cdf of the GSM model.

Author(s)

Mahdi Teimouri

References

S. Venturini, F. Dominici, and G. Parmigiani, 2008. Gamma shape mixtures for heavy-tailed distributions, *The Annals of Applied Statistics*, 2(2), 756–776.

Examples

```
data<-seq(0,20,0.1)
omega<-c(0.05, 0.1, 0.15, 0.2, 0.25, 0.25)
beta<-2
pgsm(data, omega, beta)
```

pjsb

Computing the cumulative distribution function of Johnson's SB (JSB) distribution

Description

Computes the cumulative distribution function of the four-parameter JSB distribution given by

$$F(x|\Theta) = \int_{\xi}^x \frac{\delta\lambda}{\sqrt{2\pi}(u-\xi)(\lambda+\xi-u)} \exp\left\{-\frac{1}{2}\left[\gamma + \delta \log\left(\frac{u-\xi}{\lambda+\xi-u}\right)\right]^2\right\} du,$$

where $\xi < x < \lambda + \xi$, $\Theta = (\delta, \gamma, \lambda, \xi)^T$ with $\delta, \lambda > 0$, $-\infty < \gamma < \infty$, and $-\infty < \xi < \infty$.

Usage

```
pjsb(data, param, log.p = FALSE, lower.tail = TRUE)
```

Arguments

<code>data</code>	Vector of observations.
<code>param</code>	Vector of the parameters δ , γ , λ , and ξ .
<code>log.p</code>	If TRUE, then <code>log(cdf)</code> is returned.
<code>lower.tail</code>	If FALSE, then <code>1-cdf</code> is returned.

Value

A vector of length `n`, giving random generated values from JSB distribution.

Author(s)

Mahdi Teimouri

Examples

```
data<-rnorm(10)
param<-c(delta<-1, gamma<-3, lambda<-12, xi<-5)
pjsb(data, param, log.p = FALSE, lower.tail = TRUE)
```

pmixture	<i>Computing cumulative distribution function of the well-known mixture models</i>
----------	--

Description

Computes cumulative distribution function (cdf) of the mixture model. The general form for the cdf of the mixture model is given by

$$F(x, \Theta) = \sum_{j=1}^K \omega_j F(x, \theta_j),$$

where $\Theta = (\theta_1, \dots, \theta_K)^T$, is the whole parameter vector, θ_j for $j = 1, \dots, K$ is the parameter space of the j -th component, i.e. $\theta_j = (\alpha_j, \beta_j)^T$, $F_j(\cdot, \theta_j)$ is the cdf of the j -th component, and known constant K is the number of components. The vector of mixing parameters is given by $\omega = (\omega_1, \dots, \omega_K)^T$ where ω_j s sum to one, i.e., $\sum_{j=1}^K \omega_j = 1$. Parameters α and β are the shape and scale parameters or both are the shape parameters. In the latter case, the parameters α and β are called the first and second shape parameters, respectively. The families considered for each component include Birnbaum-Saunders, Burr type XII, Chen, F, Frechet, Gamma, Gompertz, Log-normal, Log-logistic, Lomax, skew-normal, and Weibull.

Usage

```
pmixture(data, g, K, param)
```

Arguments

data	Vector of observations.
g	Name of the family including: "birnbaum-saunders", "burrxii", "chen", "f", "frechet", "gamma", "gompertz", "log-normal", "log-logistic", "lomax", "skew-normal", and "weibull".
K	Number of components.
param	Vector of the ω , α , β , and λ .

Details

For the skew-normal case, α , β , and λ are the location, scale, and skewness parameters, respectively.

Value

A vector of the same length as data, giving the cdf of the mixture model computed at data.

Author(s)

Mahdi Teimouri

Examples

```
data<-seq(0,20,0.1)
K<-2
weight<-c(0.6,0.4)
alpha<-c(1,2)
beta<-c(2,1)
param<-c(weight,alpha,beta)
pmixture(data, "weibull", K, param)
```

 rgsm

Simulating realizations from the gamma shape mixture model

Description

Simulates realizations from a gamma shape mixture (GSM) model with probability density function given by

$$f(x, \Theta) = \sum_{j=1}^K \omega_j \frac{\beta^j}{\Gamma(j)} x^{j-1} \exp(-\beta x),$$

where $\Theta = (\omega_1, \dots, \omega_K, \beta)^T$ is the parameter vector and known constant K is the number of components. The vector of mixing parameters is given by $\omega = (\omega_1, \dots, \omega_K)^T$ where ω_j s sum to one, i.e., $\sum_{j=1}^K \omega_j = 1$. Here β is the rate parameter that is equal for all components.

Usage

```
rgsm(n, omega, beta)
```


Arguments

n	Number of requested random realizations.
omega	Vector of the mixing parameters.
beta	The rate parameter.

Value

A vector of length n, giving random generated values from GSM model.

Author(s)

Mahdi Teimouri

References

S. Venturini, F. Dominici, and G. Parmigiani, 2008. Gamma shape mixtures for heavy-tailed distributions, *The Annals of Applied Statistics*, 2(2), 756–776.

Examples

```
n<-100
omega<-c(0.05, 0.1, 0.15, 0.2, 0.25, 0.25)
beta<-2
rgsm(n, omega, beta)
```

rjsb

Simulating realizations from the Johnson's SB (JSB) distribution

Description

Simulates realizations from four-parameter JSB distribution with probability density function given by

$$f(x|\Theta) = \frac{\delta\lambda}{\sqrt{2\pi}(x-\xi)(\lambda+\xi-x)} \exp\left\{-\frac{1}{2}\left[\gamma + \delta \log\left(\frac{x-\xi}{\lambda+\xi-x}\right)\right]^2\right\},$$

where $\xi < x < \lambda + \xi$, $\Theta = (\delta, \gamma, \lambda, \xi)^T$ with $\delta, \lambda > 0$, $-\infty < \gamma < \infty$, and $-\infty < \xi < \infty$.

Usage

```
rjsb(n, param)
```

Arguments

n	Number of requested random realizations.
param	Vector of the parameters δ , γ , λ , and ξ .

Value

A vector of length n , giving random generated values from JSB distribution.

Author(s)

Mahdi Teimouri

Examples

```
n<-100
param<-c(delta<-1, gamma<-3, lambda<-12, xi<-5)
rjsb(n, param)
```

 rmixture

Generating random realizations from the well-known mixture models

Description

Generates iid realizations from the mixture model with pdf given by

$$f(x, \Theta) = \sum_{j=1}^K \omega_j f(x, \theta_j),$$

where K is the number of components, θ_j , for $j = 1, \dots, K$ is parameter space of the j -th component, i.e. $\theta_j = (\alpha_j, \beta_j)^T$, and Θ is the whole parameter vector $\Theta = (\theta_1, \dots, \theta_K)^T$. Parameters α and β are the shape and scale parameters or both are the shape parameters. In the latter case, parameters α and β are called the first and second shape parameters, respectively. We note that the constants ω_j s sum to one, i.e., $\sum_{j=1}^K \omega_j = 1$. The families considered for the cdf f include Birnbaum-Saunders, Burr type XII, Chen, F, Fréchet, Gamma, Gompertz, Log-normal, Log-logistic, Lomax, skew-normal, and Weibull.

Usage

```
rmixture(n, g, K, param)
```

Arguments

n	Number of requested random realizations.
g	Name of the family including "birnbaum-saunders", "burrxii", "chen", "f", "frechet", "gamma", "gompertz", "log-normal", "log-logistic", "lomax", "skew-normal", and "weibull".
K	Number of components.
param	Vector of the ω , α , β , and λ .

Details

For the skew-normal case, α , β , and λ are the location, scale, and skewness parameters, respectively.

Value

A vector of length n , giving a sequence of random realizations from given mixture model.

Author(s)

Mahdi Teimouri

Examples

```
n<-50
K<-2
weight<-c(0.3,0.7)
alpha<-c(1,2)
beta<-c(2,1)
param<-c(weight,alpha,beta)
rmixture(n, "weibull", K, param)
```

skewtreg

Robust multiple linear regression modelling when error term follows a skew Student's t distribution

Description

Robust multiple linear regression modelling with skew Student's t error term. The density function of skew Student's t is given by

$$f(x, \Theta) = \frac{2}{\sigma} t(z; \nu) T\left(\lambda z \sqrt{\frac{\nu + 1}{\nu + z^2}}; \nu + 1\right),$$

where $z = (x - \mu)/\sigma$, $-\infty < \mu < \infty$ is the location parameter, $\sigma > 0$ is the scale parameter, and $-\infty < \lambda < \infty$ is the skewness parameter. Also, $t(u, \nu)$ and $T(u, \nu)$ denote the density and distribution functions of the Student's t distribution with ν degrees of freedom at point u , respectively. If $\lambda = 0$, then the skew Student's t distribution turns into the ordinary Student's t distribution that is symmetric around μ . Since Student's t is a heavy tailed distribution, it is so useful for regression analysis in presence of outliers.

Usage

```
skewtreg(y, x, Fisher=FALSE)
```

Arguments

y	vector of response variable.
x	vector or matrix of explanatory variable(s).
Fisher	Either TRUE or FALSE. By default Fisher==FALSE; otherwise the observed Fisher information matrix and asymptotic standard errors for estimated regression coefficients are evaluated.

Value

A list of estimated regression coefficients, asymptotic standard error, corresponding p-values, estimated parameters of error term (skew Student's t), F statistic, R-square and adjusted R-square, and observed Fisher information matrix is given.

Author(s)

Mahdi Teimouri

Examples

```
n<-20
x<-rnorm(n)
y<-2+2*x+rt(n,df=2)
skewtreg(y,x,Fisher=FALSE)
```

SW	<i>Southern loblolly pine plantation</i>
----	--

Description

Tree list from a U.S. Forest Service Forest Inventory and Analysis (FIA) plot PLT_CN 259082471010854 measured in 2011 and represents a typical loblolly pine plantation.

Usage

```
data(SW)
```

Format

A data frame containing 18 trees (rows) and two columns. Columns are the trees' scientific name and diameter at breast height in inches.

welcome	<i>Starting message when loading ForestFit</i>
---------	--

Description

It contains a welcome message for user of ForestFit.

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